

UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS

## MATHEMATICS

9709/72
Paper 7 Probability \& Statistics 2 (S2)
May/June 2013
1 hour 15 minutes

Additional Materials: | Answer Booklet/Paper |
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| Graph Paper |
| List of Formulae (MF9) |

## READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.
Answer all the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.
At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 50 .
Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.

1 It is known that $1.2 \%$ of rods made by a certain machine are bent. The random variable $X$ denotes the number of bent rods in a random sample of 400 rods.
(i) State the distribution of $X$.
(ii) State, with a reason, a suitable approximate distribution for $X$.
(iii) Use your approximate distribution to find the probability that the sample will include more than 2 bent rods.

2 A random variable $X$ has probability density function given by

$$
\mathrm{f}(x)= \begin{cases}\frac{2}{3} x & 1 \leqslant x \leqslant 2 \\ 0 & \text { otherwise }\end{cases}
$$

(i) Find $\mathrm{E}(X)$.
(ii) Find $\mathrm{P}(X<\mathrm{E}(X))$.
(iii) Hence explain whether the mean of $X$ is less than, equal to or greater than the median of $X$.

3 The heights of a certain variety of plant have been found to be normally distributed with mean 75.2 cm and standard deviation 5.7 cm . A biologist suspects that pollution in a certain region is causing the plants to be shorter than usual. He takes a random sample of $n$ plants of this variety from this region and finds that their mean height is 73.1 cm . He then carries out an appropriate hypothesis test.
(i) He finds that the value of the test statistic $z$ is -1.563 , correct to 3 decimal places. Calculate the value of $n$. State an assumption necessary for your calculation.
(ii) Use this value of the test statistic to carry out the hypothesis test at the $6 \%$ significance level.

4 The masses, in grams, of a certain type of plum are normally distributed with mean $\mu$ and variance $\sigma^{2}$. The masses, $m$ grams, of a random sample of 150 plums of this type were found and the results are summarised by $\Sigma m=9750$ and $\Sigma m^{2}=647500$.
(i) Calculate unbiased estimates of $\mu$ and $\sigma^{2}$.
(ii) Calculate a $98 \%$ confidence interval for $\mu$.

Two more random samples of plums of this type are taken and a $98 \%$ confidence interval for $\mu$ is calculated from each sample.
(iii) Find the probability that neither of these two intervals contains $\mu$.

5 Packets of cereal are packed in boxes, each containing 6 packets. The masses of the packets are normally distributed with mean 510 g and standard deviation 12 g . The masses of the empty boxes are normally distributed with mean 70 g and standard deviation 4 g .
(i) Find the probability that the total mass of a full box containing 6 packets is between 3050 g and 3150 g .
(ii) A packet and an empty box are chosen at random. Find the probability that the mass of the packet is at least 8 times the mass of the empty box.

6 The number of cases of asthma per month at a clinic has a Poisson distribution. In the past the mean has been 5.3 cases per month. A new treatment is introduced. In order to test at the $5 \%$ significance level whether the mean has decreased, the number of cases in a randomly chosen month is noted.
(i) Find the critical region for the test and, given that the number of cases is 2 , carry out the test.
(ii) Explain the meaning of a Type I error in this context and state the probability of a Type I error.
(iii) At another clinic the mean number of cases of asthma per month has the independent distribution $\operatorname{Po}(13.1)$. Assuming that the mean for the first clinic is still 5.3, use a suitable approximating distribution to estimate the probability that the total number of cases in the two clinics in a particular month is more than 20.

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